Theory of Object Class Uncertainty and its Application

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References

[1] P. K. Saha and J. K. Udupa, "Optimum image thresholding via class uncertainty and region homogeneity," IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 23, pp. 689-706, 2001.

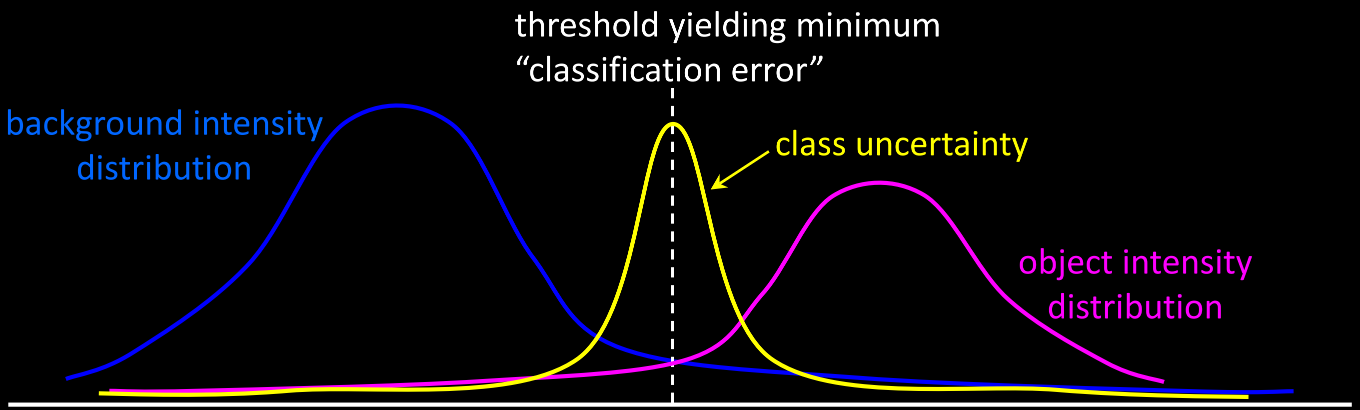
[2] P. K. Saha, B. Das, and F. W. Wehrli, "An object class-uncertainty induced adaptive force and its application to a new hybrid snake," Pattern Recognition, vol. 40, pp. 2656-2671, 2007.

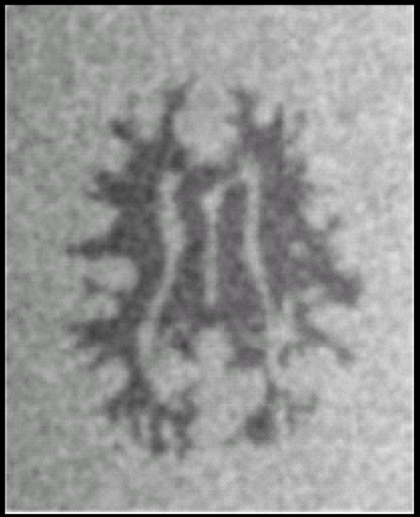
[3] Y. Liu, G. Liang, and P. K. Saha, "A new multi-object image thresholding method based on correlation between object class uncertainty and intensity gradient," Medical physics, vol. 39, pp. 514-532, 2012.

Outline

- Theory Of Object Class Uncertainty
- Applications To Optimum Thresholding
- Applications To Snake

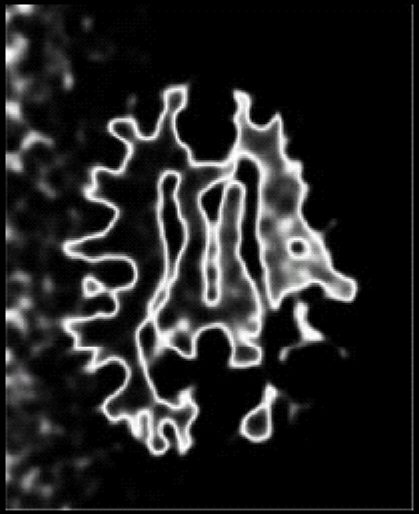
Object Class Uncertainty





Gray-scale image

Postulate. In an image with fuzzy boundaries, at optimum partitioning of object classes, voxels with high class uncertainty appear in the vicinity of object boundaries.



Class uncertainty image at optimum thresholding

- Saha, Udupa, "Optimum image thresholding via class uncertainty and region homogeneity," IEEE Trans Patt Anal Mach Intell, **23**: 689-706, 2001

Computation of Object Class Uncertainty

A *priori* probability an object pixel having intensity g

$$p_O(g) = P(f(c) = g \mid c \in F_O),$$

where P represents "probability," and F_O represents the true object class

A *priori* probability a background pixel having intensity g

$$p_B(g) = P(f(c) = g \mid c \in F_B),$$

where F_B represents the true object class

θ : *A priori* probability of any pixel belonging to object

$p(g)$: *A priori* probability of any pixel having intensity g

$$p(g) = \theta p_O(g) + (1 - \theta) p_B(g).$$

Computation of Object Class Uncertainty

A posteriori probability:

$$P(c \in F_O \,|f(c) = g) = \frac{\theta p_O(g)}{p(g)}$$

$$P(c \in F_B \,|f(c) = g) = \frac{(1-\theta)p_B(g)}{p(g)}$$

$h(g)$: "object class uncertainty" at intensity g

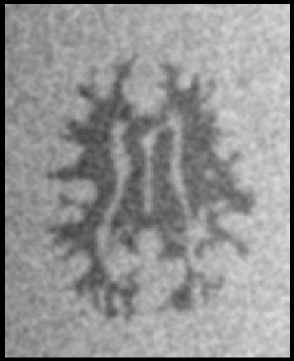
$$h(g) = -\frac{\theta p_O(g)}{p(g)}\log\frac{\theta p_O(g)}{p(g)} - \frac{(1-\theta)p_B(g)}{p(g)}\log\frac{(1-\theta)p_B(g)}{p(g)}$$

Optimum Thresholding

Postulate. In an image with fuzzy boundaries, at optimum partitioning of object classes, voxels with high class uncertainty appear in the vicinity of object boundaries.

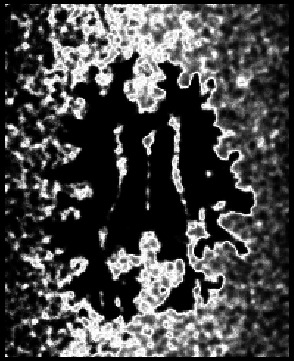
$$E(t) = \sum_{c \in C} H_t\big(f(c)\big)\big(1 - \Delta_{\mathrm{rank}}(c)\big) + \Big(1 - H_t\big(f(c)\big)\Big)\Delta_{\mathrm{rank}}(c),$$

H_t is the uncertainty map at a threshold t and Δ_{rank} is a rank-normalized gradient operator

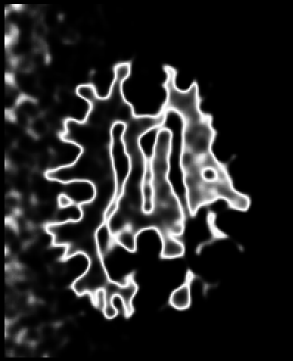


original image

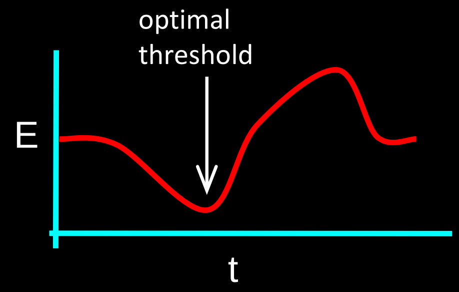
gradient image



uncertainty image at a non-optimal threshold



uncertainty image at an optimal threshold



A theory to combine information theoretic measures with image gradient features

- Saha, Udupa, "Optimum image thresholding via class uncertainty and region homogeneity," IEEE Trans Patt Anal Mach Intell, **23**: 689-706, 2001

Rank-Normalized Gradient

- DoG measures are sensitive to the standard deviation parameter of the normalizing Gaussian function
- Rank-based normalization of the gradient parameter
  - A parameter-free approach of normalization

$$\Delta_{\text{rank}} = \frac{LC\big(\Delta(c)\big)}{LC\left(\Delta_{\max}\right)},$$

where

- Δ is the intensity gradient operator
- $LC(x) = \sum_{y \le x} L(y)$, and $L(y)$ is the histogram count for the intensity gradient value y

- Saha, Udupa, "Optimum image thresholding via class uncertainty and region homogeneity," IEEE Trans Patt Anal Mach Intell, **23**: 689-706, 2001

Optimum Thresholding Algorithm

Principle. Minimization of Uncertainty Homogeneity Energy (MHUE) $E(t)$

$$t_{\text{opt}} = \arg\min_t E(t)$$

An efficient computation of the Energy function $E(t)$

$$E(t) = \sum_{c \in C} H_t(f(c))\left(1 - \Delta_{\text{rank}}(c)\right) + \left(1 - H_t(f(c))\right)\Delta_{\text{rank}}(c)$$

$\forall$ intensity value i,

$$X(i) = \sum_{c \in C \,|\, f(c)=i} 1 - \Delta_{\text{rank}},$$

$$Y(i) = \sum_{c \in C \,|\, f(c)=i} \Delta_{\text{rank}},$$

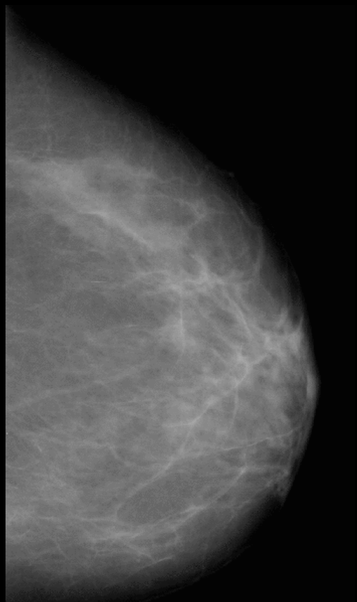
Efficient formulation of $E(t)$

$$E(t) = \sum_i H_t(i)X(i) + \left(1 - H_t(i)\right)Y(i)$$

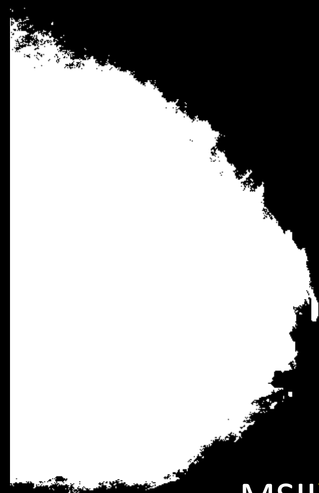
Note: Number of possible intensity values in an image is far less than the number of pixels/voxels in the image

- Saha, Udupa, "Optimum image thresholding via class uncertainty and region homogeneity," IEEE Trans Patt Anal Mach Intell, **23**: 689-706, 2001

Results

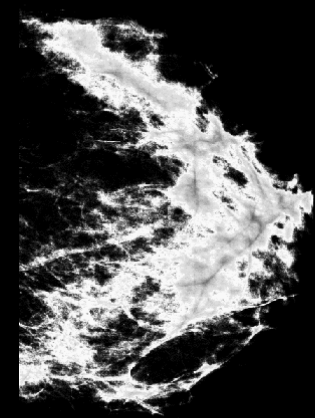


Digitized Mammogram



MSII†

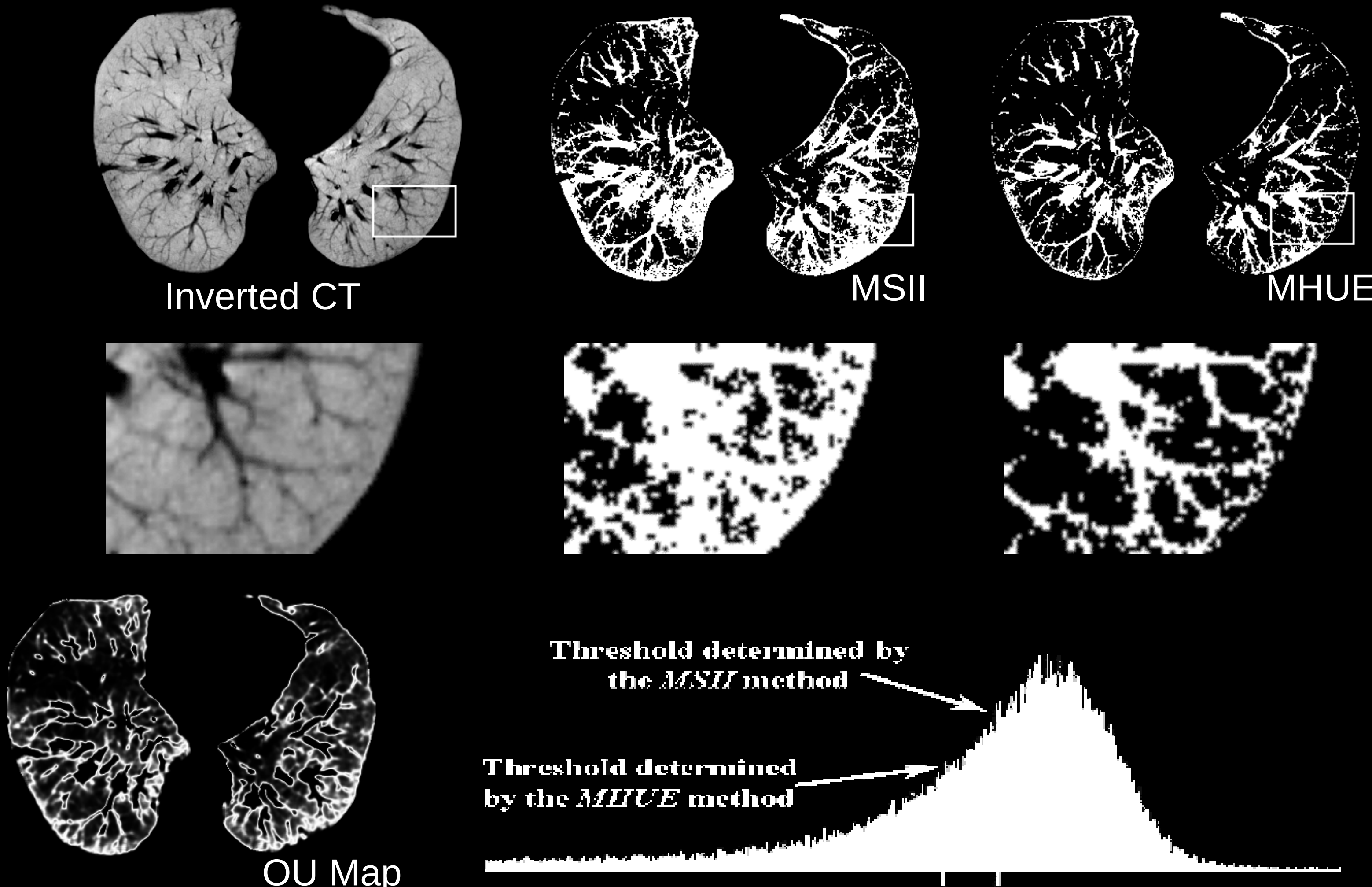
MHUE



optimum uncertainty (OU) map

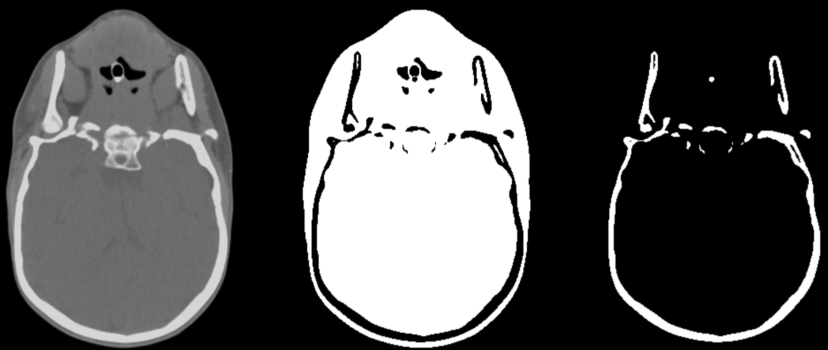
†Leung, Lam, "Maximum segmented image information thresholding," Graph Mod Imag Proc, **60**: 57-76, 1998

Results



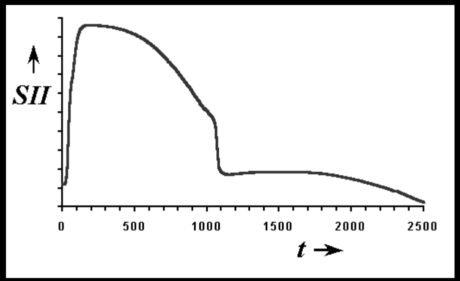
Inverted CT
MSII
MHUE
OU Map
Threshold determined by the MSII method
Threshold determined by the MHUE method

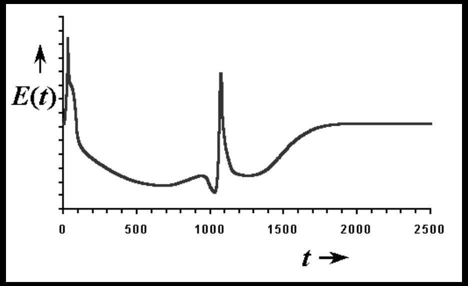
Multiple Object Segmentation

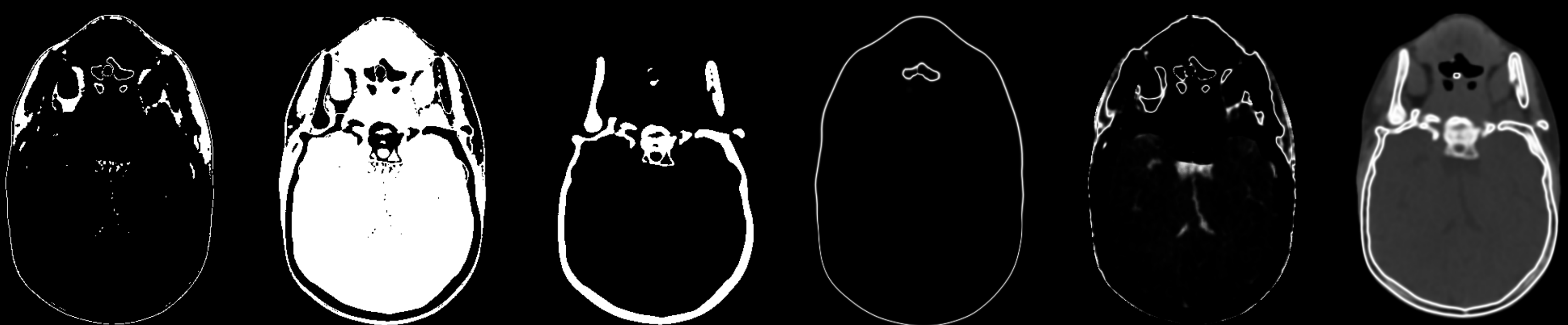


CT Slice

MSII



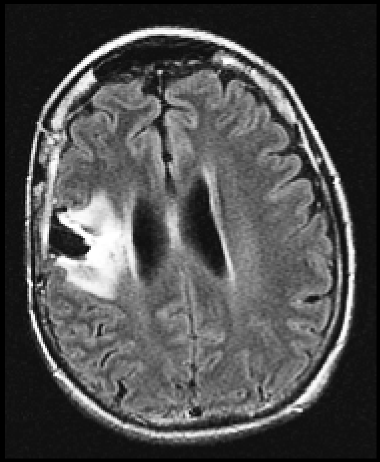




MHUE

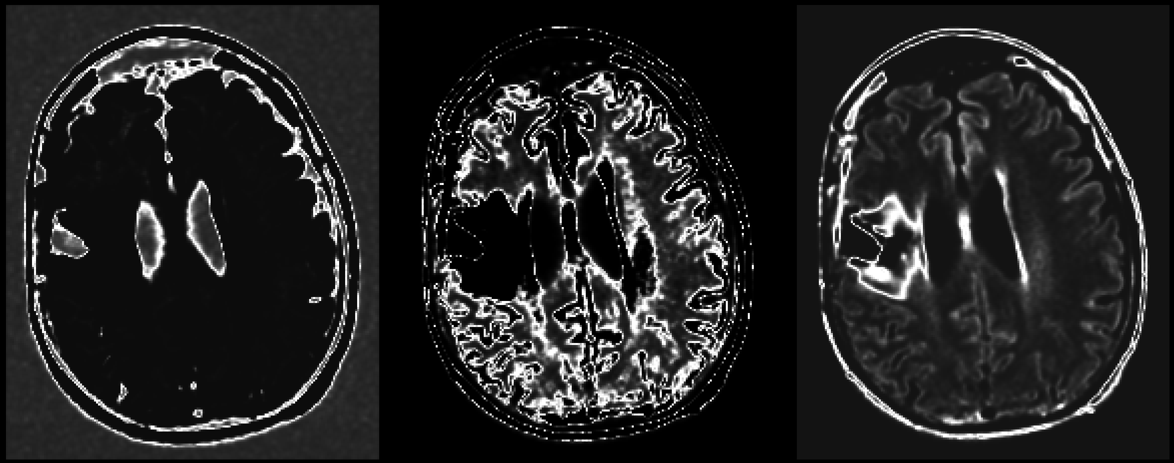
OU Map

Application on MR Slice Data

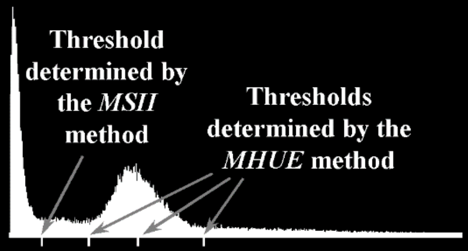


Flair

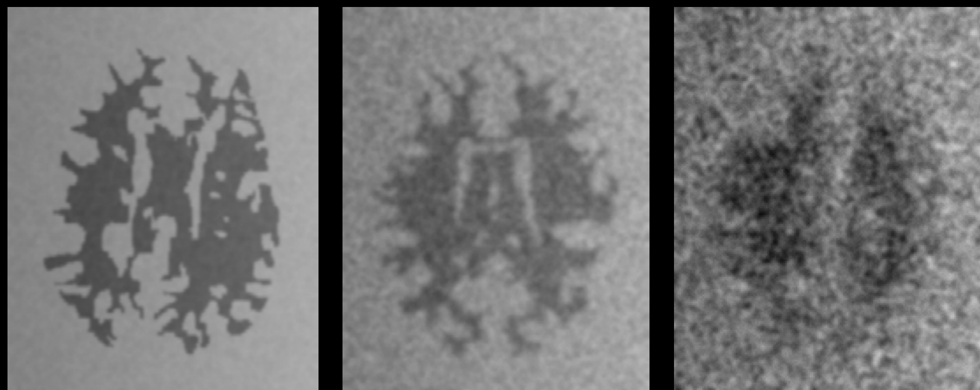
MHUE segmentation



OU Map

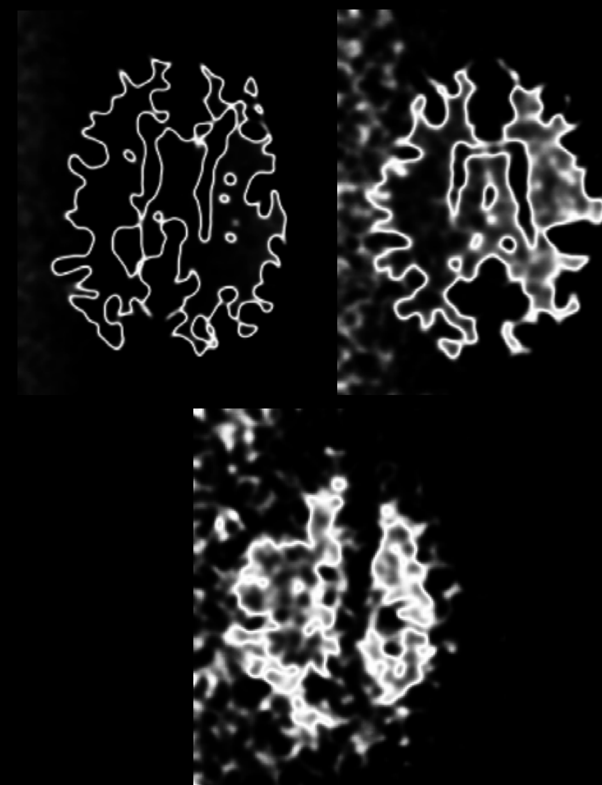


Phantom Experiment



Phantoms

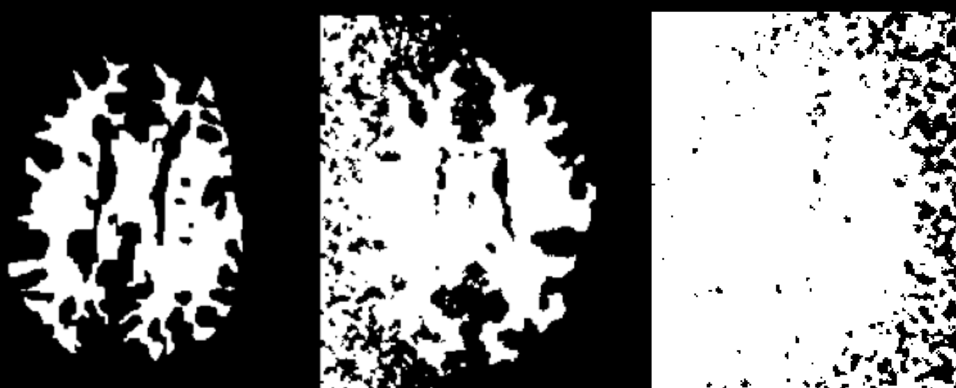
MHUE



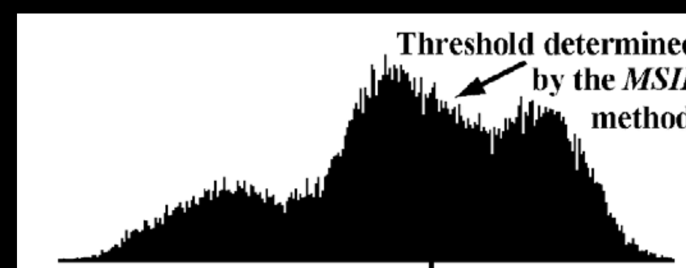
OU Map

Phantom Experiment

Phantoms



MSII



Application of Object Class Uncertainty to Snake

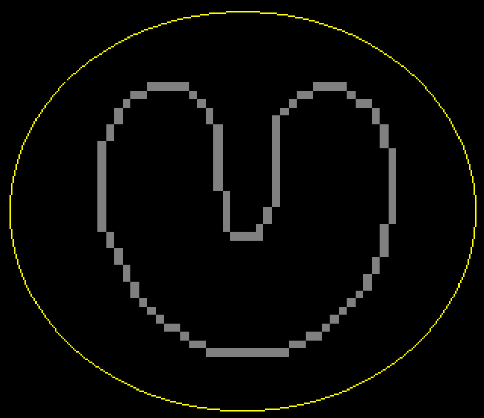
- Saha, Das, Wehrli, "An object class-uncertainty induced adaptive force and its application to a new hybrid snake," Patt Recog, **40**: 2656-2671, 2007

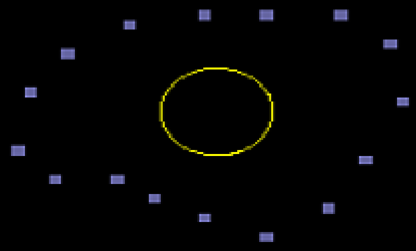
Outline

- Brief Overview of Snake
- Basic Challenges
- Object Feature Force
- *Object Class Uncertainty*
- Smart Force
- Smart Snake – Methods and Design
- Experimental Results

Curves in Motion

- Initialization
  - Squeezing Snake: Object contained entirely inside the region enclosed by the initial contour
  - Expanding Snake: Object entirely includes the region enclosed by the initial contour
  - Automatic
    - Expand from a seed point using balloon force
    - Converge from the boundary of image frame

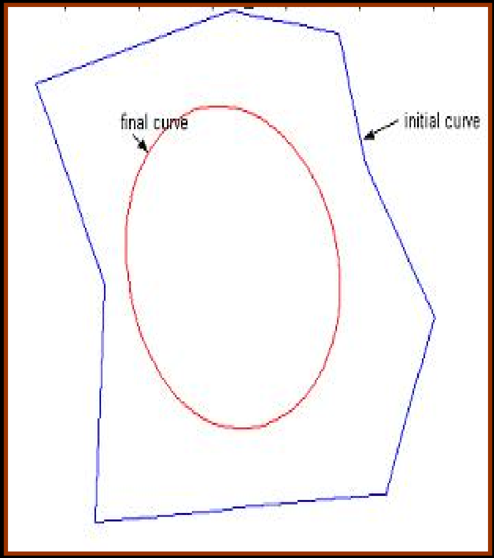




Internal Energy

The spline properties

- An elastic rubber band possessing elastic potential energy
  - Responsible for shrinking of the contour
- Behaves like a thin metal strip giving rise to bending energy
  - Bending energy is minimum for a circle.
- Total internal energy of the snake ν can be defined as

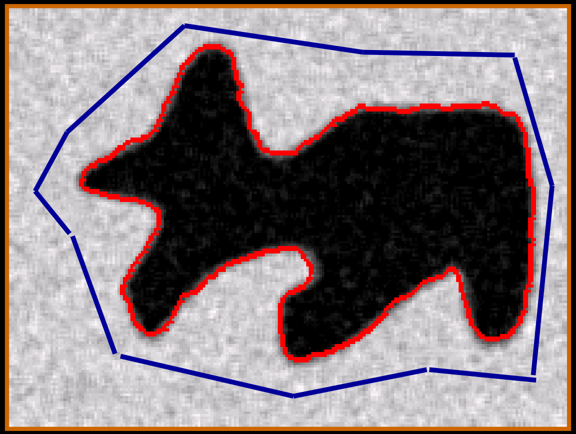


$$E_{\text{int}} = \alpha(s)\left\|\frac{\partial \nu(s)}{\partial s}\right\| + \beta(s)\left\|\frac{\partial^2 \nu(s)}{\partial s^2}\right\|^2$$

Snake: Basic Formulation

- Snake: a deformable spline ν[†]
- Basic Snake Equation

$$E_{\text{snake}} = E_{\text{int}} + E_{\text{image}} + E_{\text{con}}$$



- Internal Energy
  - String (elastic) Force
  - Rigidity Force

$$E_{\text{int}} = \alpha(s)\left\|\frac{\partial \nu(s)}{\partial s}\right\| + \beta(s)\left\|\frac{\partial^2 \nu(s)}{\partial s^2}\right\|^2$$

- Image Energy
  - Gradient
  - Intensity

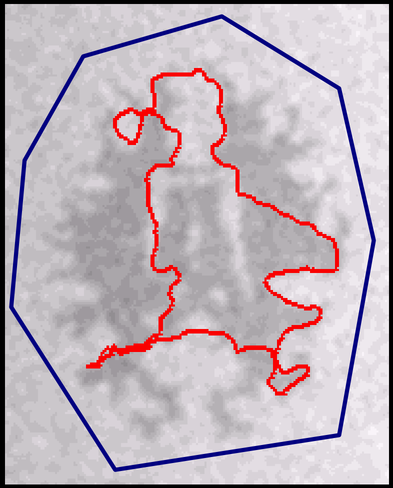
† Kass, Witkin, and Terzopoulos, "Snakes: Active Contour Models", *Int. J. Comput. Vis.*, **1,** 321-331, 1988

An Overlooked Territory

- Theory and algorithms to optimally fit in *a priori* object/background feature information

Attempts to overcome this limitation

- A blind balloon force[†] to move the snake in homogeneous regions
- Failure to arrest uncontrolled snake propagation once leaked through a weak boundary zone
- Sub optimal performance near boundaries with narrow concavities



Result using balloon snake

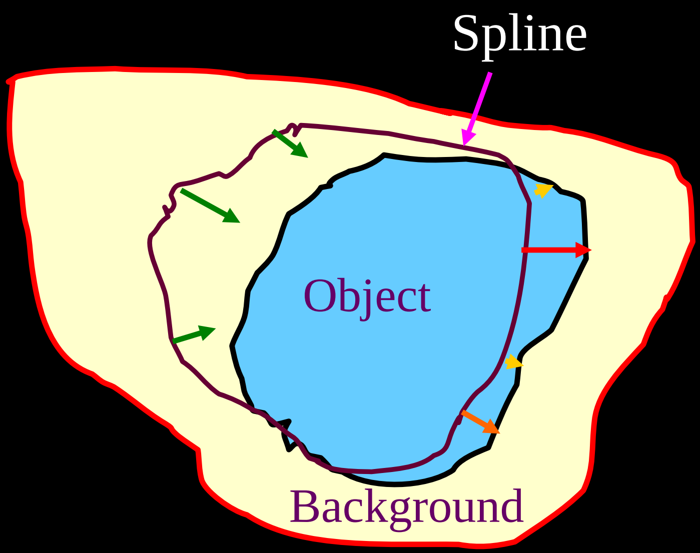
[†] Cohen and Cohen, *IEEE Trans. PAMI*, **15**, 1131-1147, 1993

Main Contribution

- Introduction of object/background feature based SMART FORCE into snake

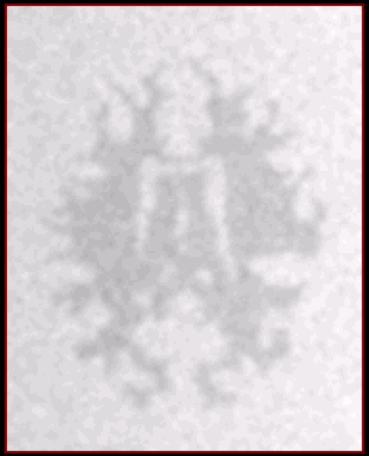
Nature of smart force

- Expanding within the object
- Compressing inside the background
- Weakens at the vicinity of the object-background interface

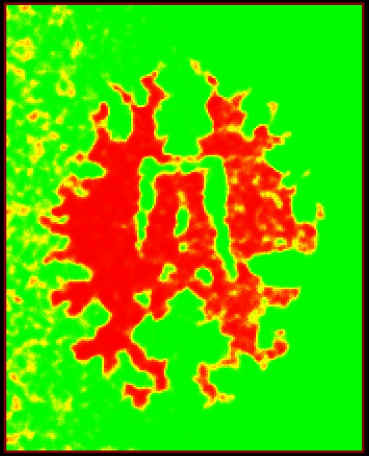


Design of the Smart Force

- Probably, we need…
  - Optimum object-background classification
  - Confidence level of the classification
- We have used …
  - Object Class Uncertainty† Based Smart Force



Gray-scale image



Smart force
- expanding
- contracting
- weak

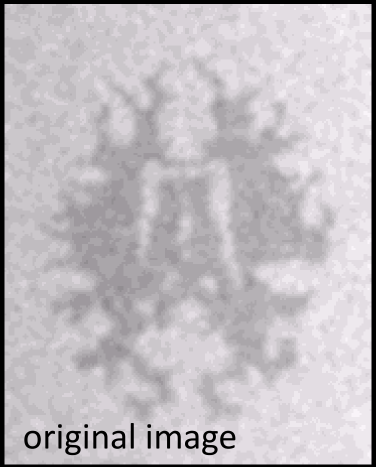
† Saha, Udupa, "Optimum image thresholding via class uncertainty and region homogeneity," IEEE Trans Patt Anal Mach Intell, **23**: 689-706, 2001

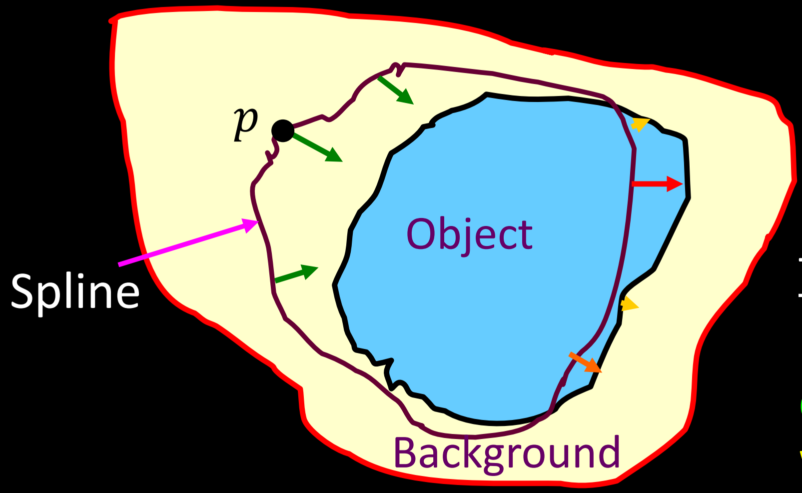
Object Class Uncertainty Induced Smart Force

$\boldsymbol{\tau}_\nu(p)$: unit vector radially outward at the location p on the contour ν

Smart force

$$F_{\text{smart},\nu}(p) = \begin{cases} 1 - h\big(f(p)\big)\boldsymbol{\tau}_\nu(p), & \text{if } p \in O, \\ -1 + h\big(f(p)\big)\boldsymbol{\tau}_\nu(p), & \text{otherwise.} \end{cases}$$



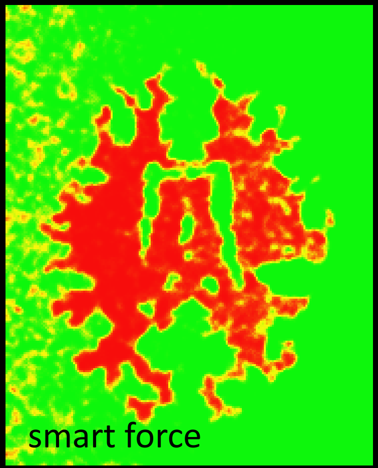


Type of Smart Forces

Expanding (inside object)

Contracting (inside background)

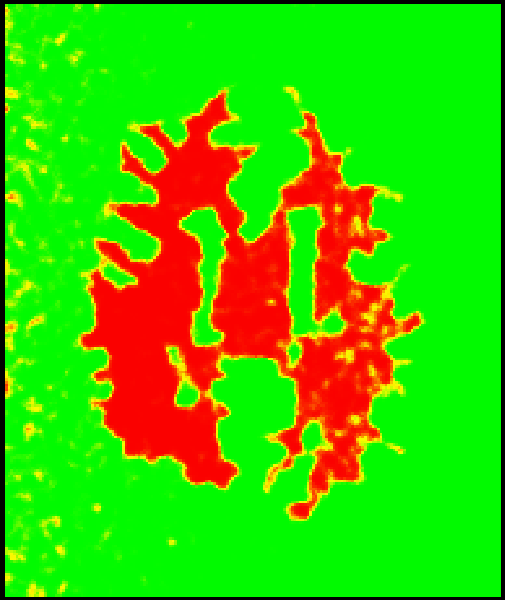
Weak force (at interface)



- Saha, Das, Wehrli, "An object class-uncertainty induced adaptive force and its application to a new hybrid snake," Patt Recog, **40**: 2656-2671, 2007

Properties of Smart Force

- Direction adaptive
  - Expands inside the object
  - Compresses within background
  - Resists uncontrolled post-leaking propagation
- Optimal response to the chaos in acquired signal
- Complementary with Image Gradient force
  - stronger inside homogeneous regions
  - weak near boundaries



smart force

Estimation of Uncertainty Force

- Prior Information about object and background intensity distribution acquired

m_O : Object mean

m_B : Background mean

σ_O : Object standard deviation

σ_B : Background standard deviation

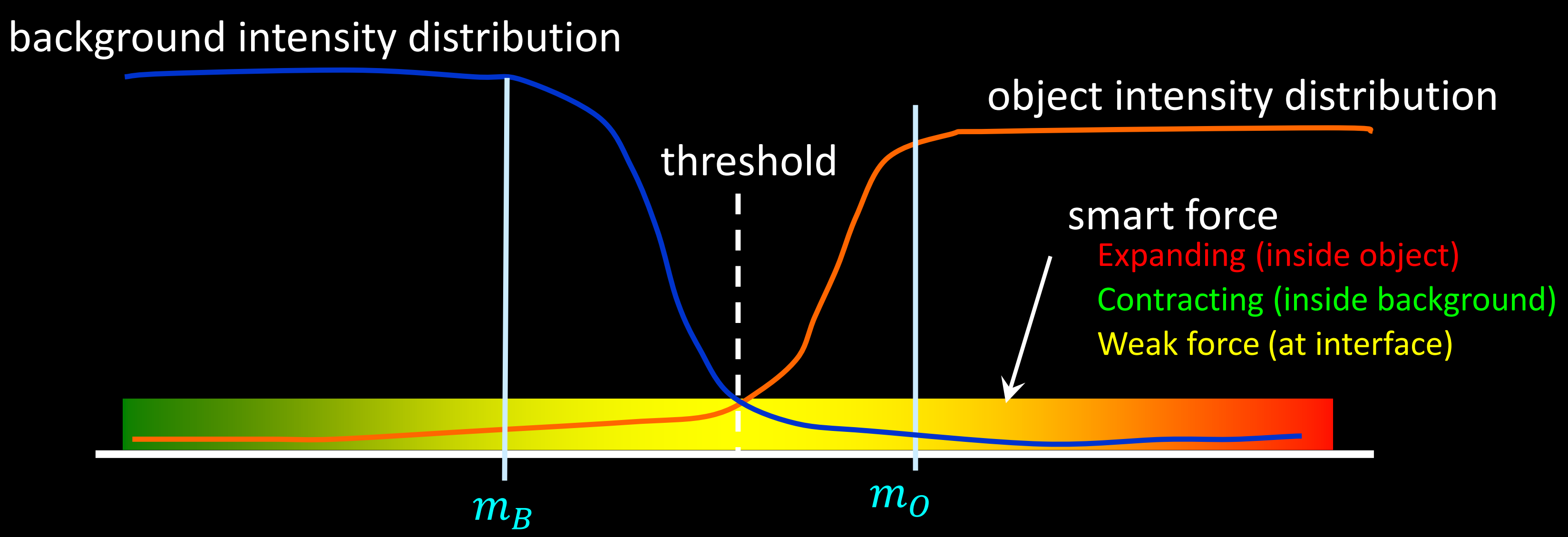
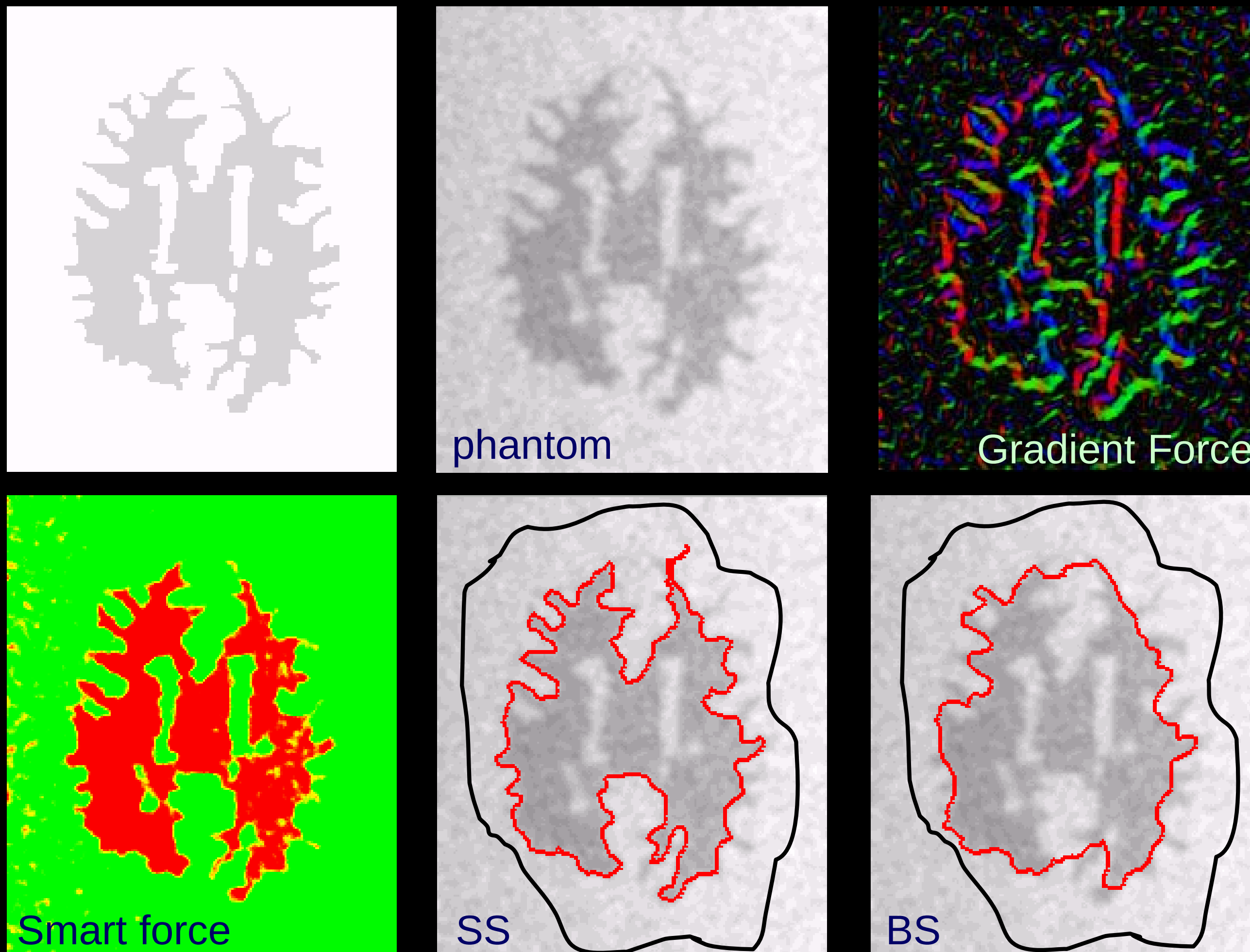


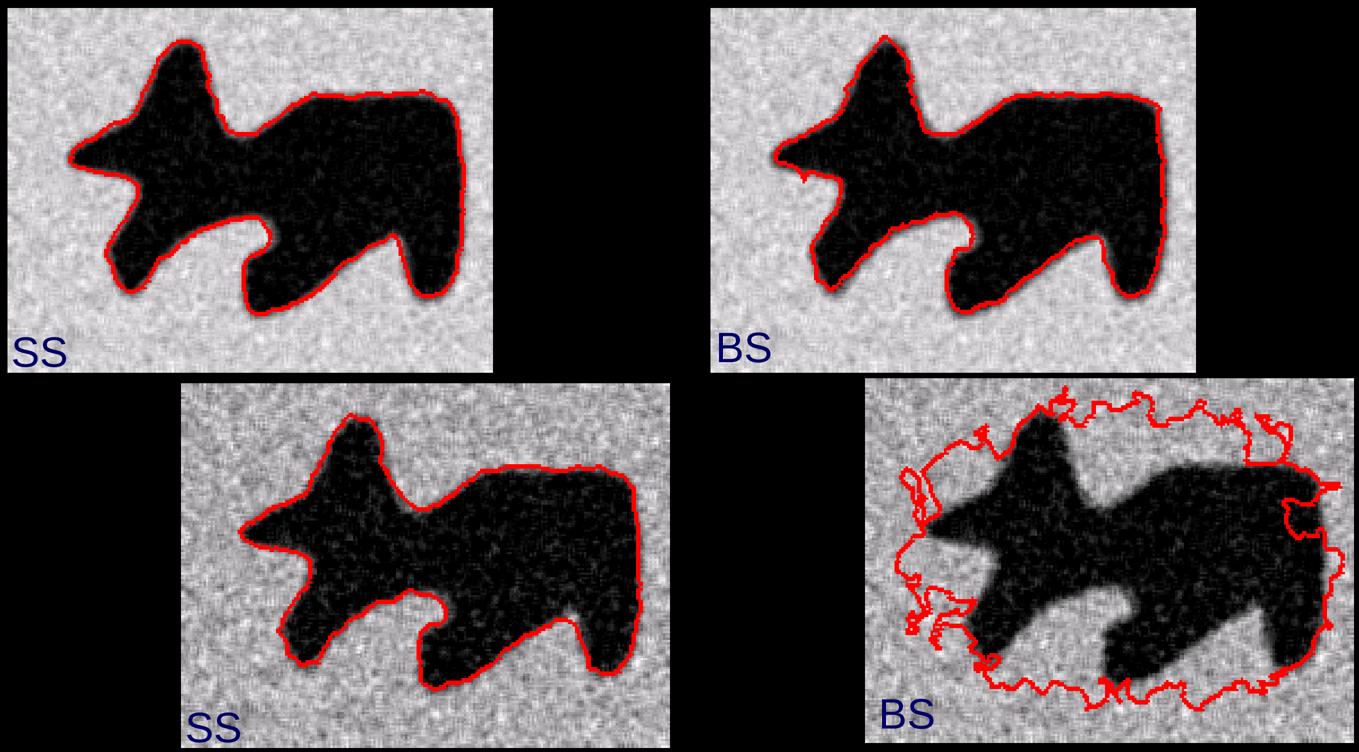
Image Force Field and Snake



Smart snake (SS)

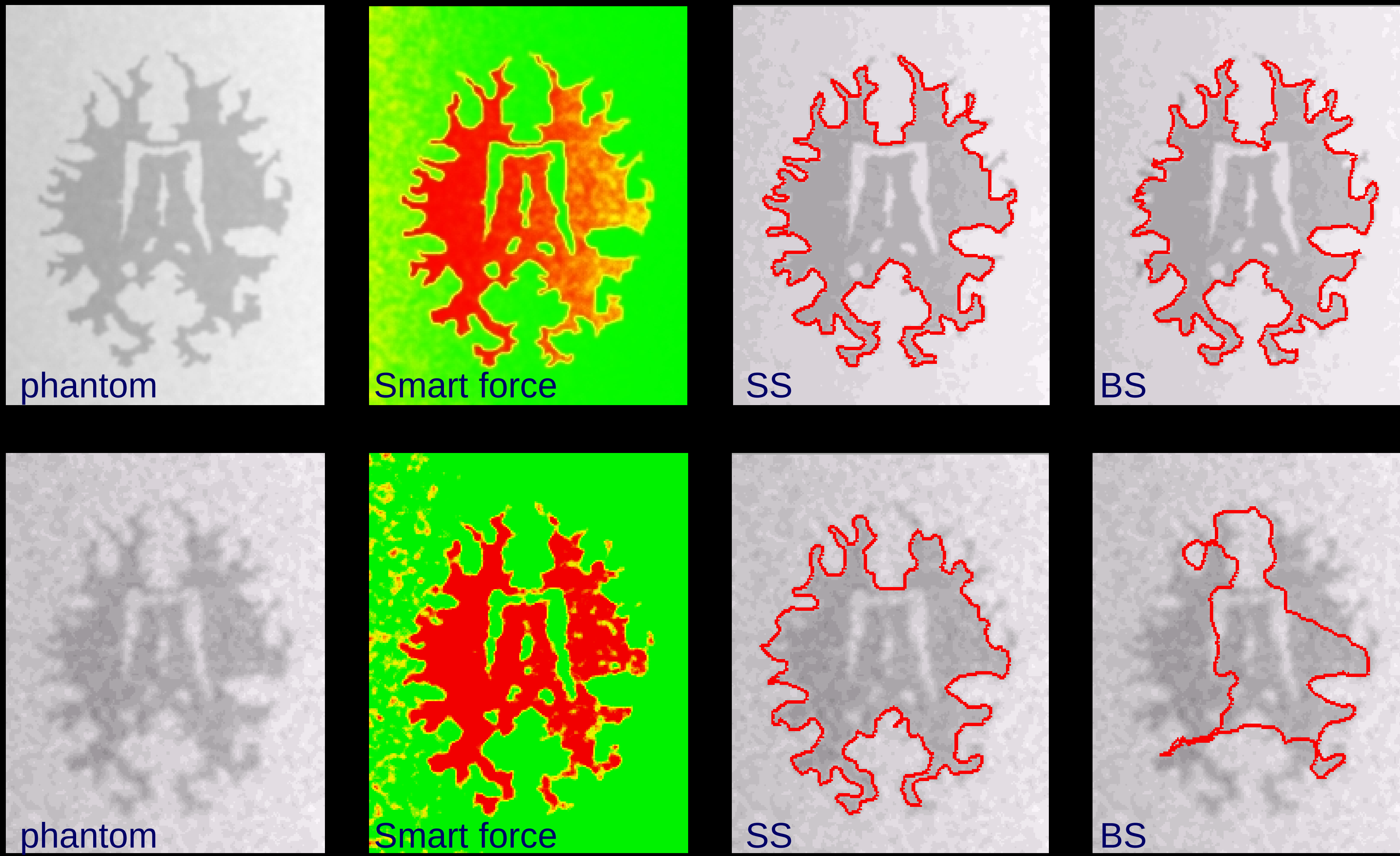
Balloon snake (BS)

Comparative Results

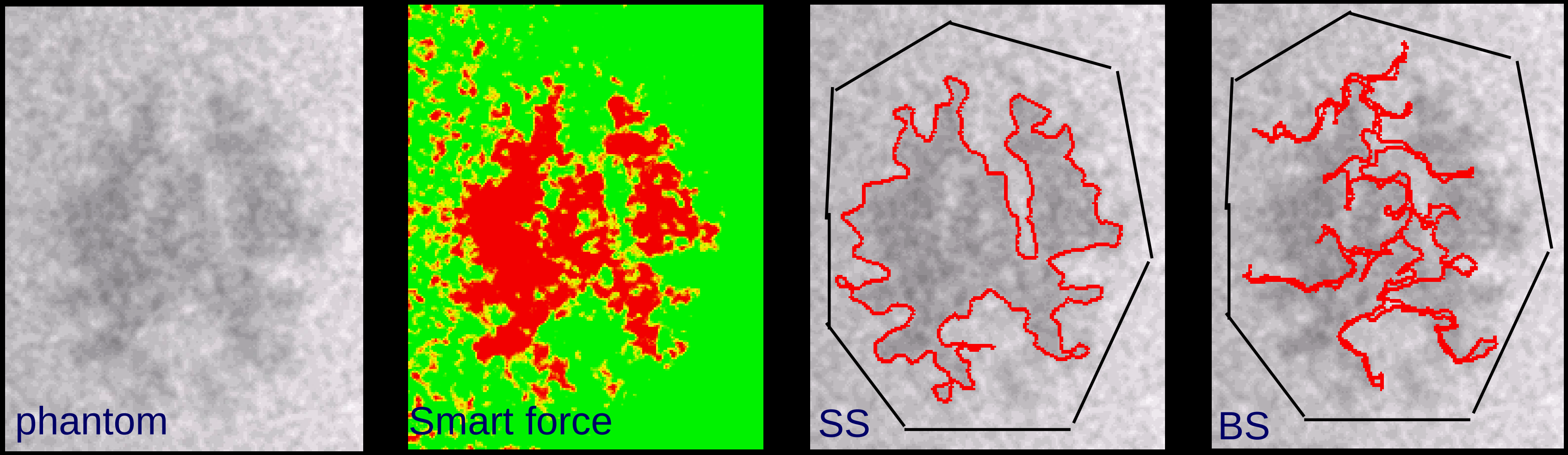


Phantom with high object-background contrast at different levels of noise and blurring

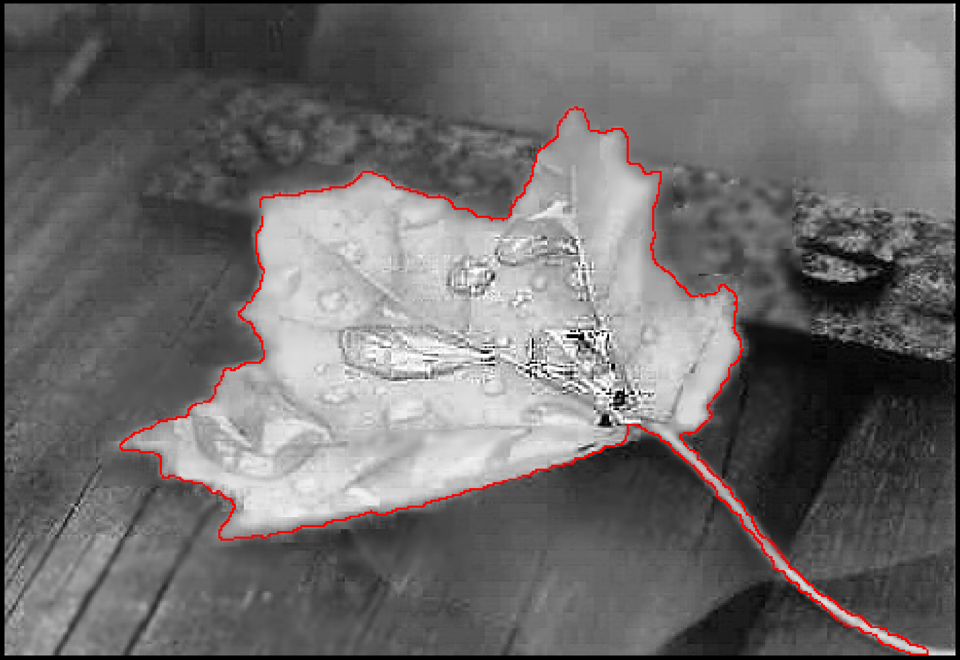
Comparative Results

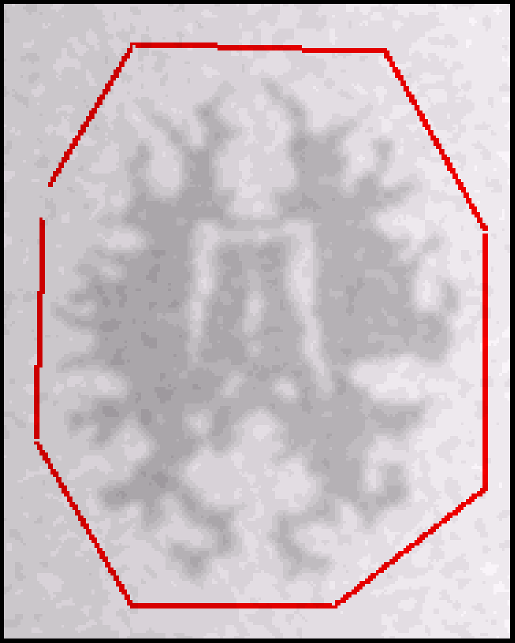


Comparative Results



Object Class Uncertainty Induced Smart Snake

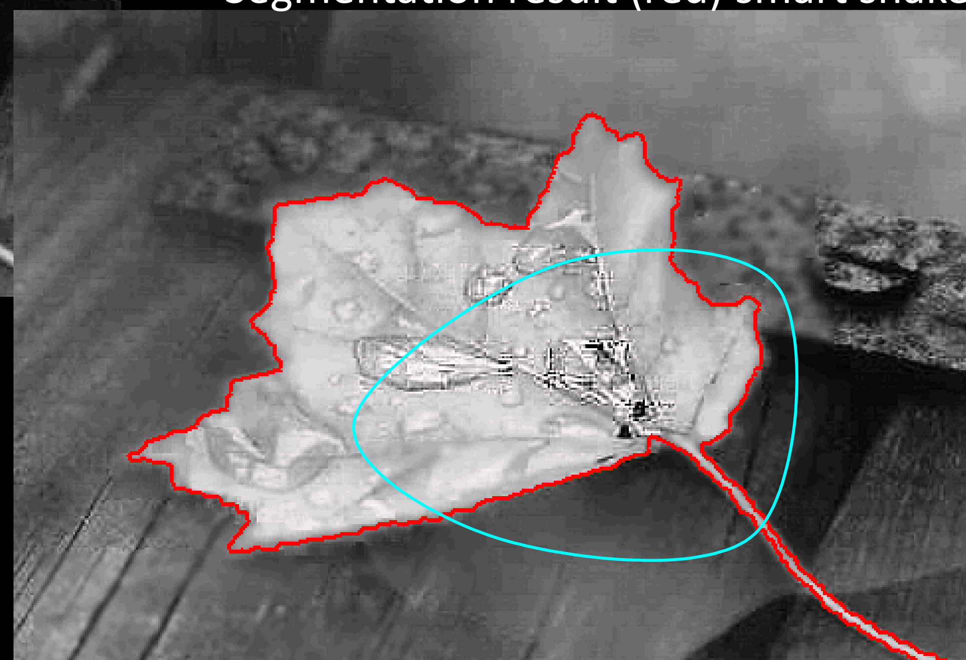




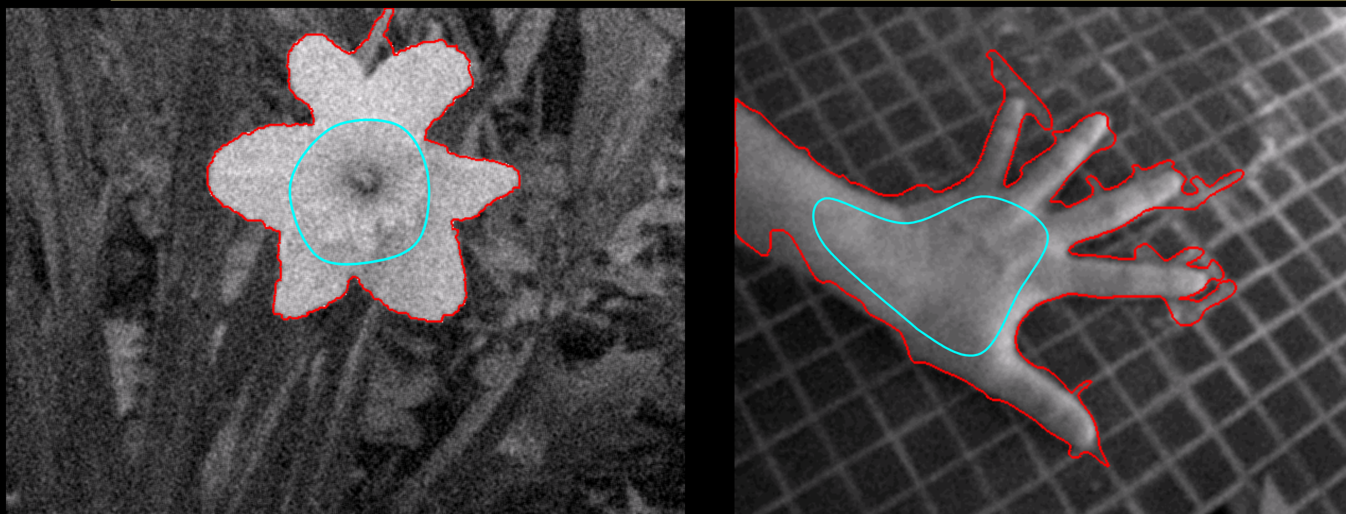
Comparison with Balloon Snake

Segmentation result (red) using balloon snake

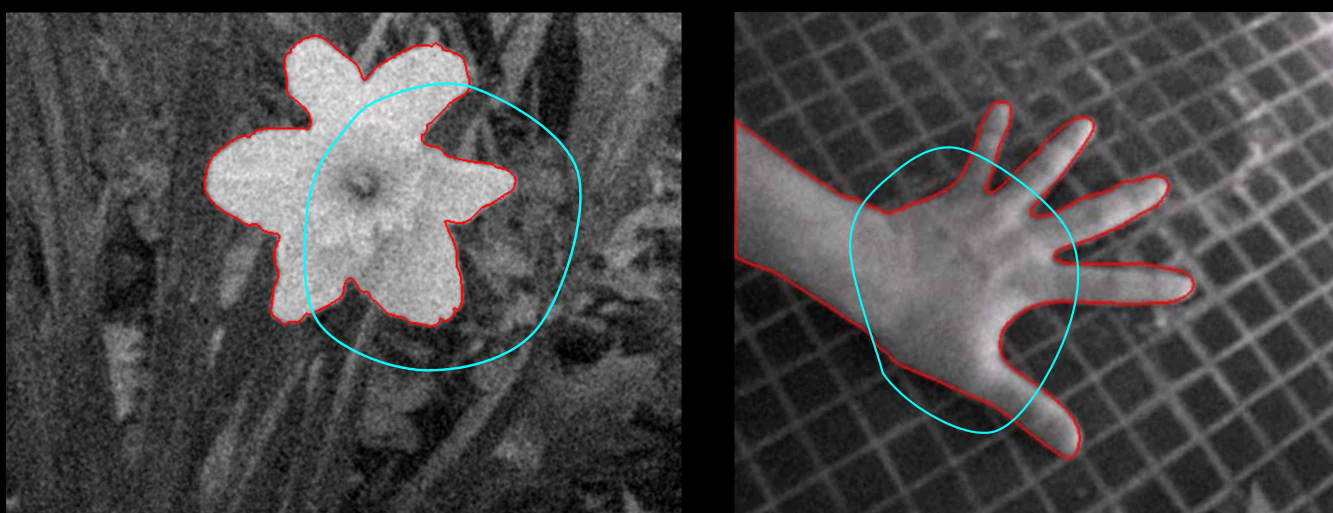
Segmentation result (red) smart snake



Comparison with Balloon Snake

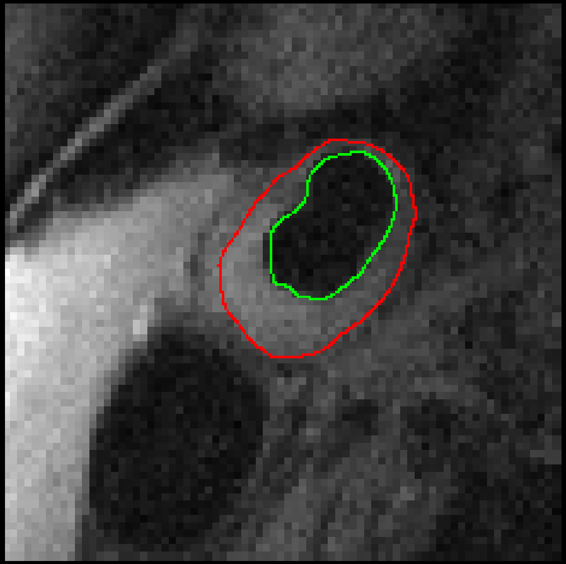


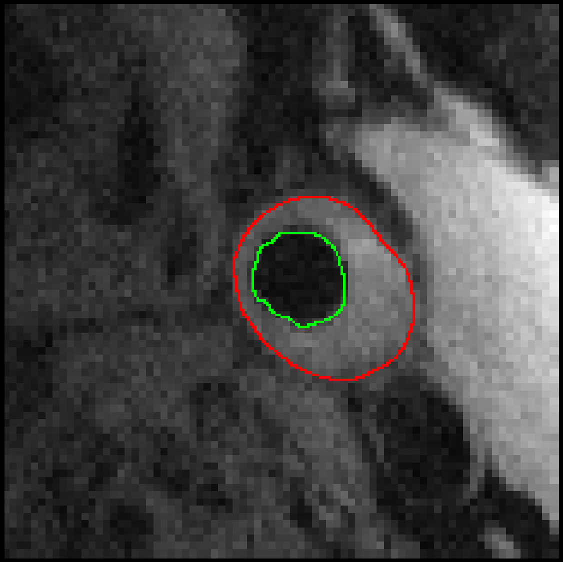
Result (red) using balloon snake



Results (red) using smart snake

Carotid Data Segmentation using Smart Snake





Summary

- Introduced object class uncertainty theory
  - Combines information theoretic measure with image features

- A fundamental postulate is stated
  - In most real life imaging applications, under optimum classification, image elements with the maximum class uncertainty appear in the vicinity of object boundaries.
  - Supported by results of application on several real images and 250 computer generated realistic phantoms
  - Potential application in multiple image and data classification tasks

Summary (Contd.)

- Application to optimum thresholding
  - Potential application in local threshold selection
  - Results of application using both real and phantom data
- Introduced object class uncertainty based smart force into snake model
  - Direction adaptive
  - Strength adaptive to fit with the inherent chaos in signal
  - Acts in complementary fashion with image gradient information
- Preliminary results of application of class uncertainty based smart snake on several natural and medical data